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Abstract

We propose a jump robust positive semidefinite rank-based estimator for the daily covariance matrix using intraday returns. It disentangles covariance estimation into variance and correlation components, allowing to estimate correlations over lower sampling frequencies to account for non-synchronous trading. The efficiency gain of disentangling covariance estimation and the jump robustness of the estimator are illustrated in a simulation study. In an application to the Dow Jones Industrial Average constituents, we show that the proposed estimator leads to more stable portfolios with a lower risk.

Keywords: Epps effect, High frequency data, Integrated covariance, Jumps, Non-synchronous trading, Realized covariance.

JEL classification: C13, C15, C32, G11.

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1 Introduction

The last decade, covolatility estimation has taken a great step forward by the development of estimation methods based on high-frequency data. Multivariate volatility estimation is characterized by two additional challenges compared to the univariate case: the asynchronicity of observations and the positive semidefiniteness of the covariance matrix estimator. The stand of the literature today is that either each element of the covariance matrix should be estimated at its own optimal sampling scheme (Hayashi and Yoshida, 2005; Zhang, 2009) or that the same sampling grid is imposed on all elements (Barndorff-Nielsen et al., 2010; De Pooter et al., 2008). In the first case, the resulting covariance matrix estimate is no longer guaranteed to be positive semidefinite, while in the second case the elements of the covariance matrix are not estimated at their optimal sampling frequency. We consider an alternative strategy. By disentangling covariance estimation into variance and correlation components, it is possible to estimate the variances at their optimal sampling frequency. Positive semidefiniteness of the covariance matrix is preserved by estimating correlations on the same sampling grid.

The proposed estimator, called the Gaussian Rank Covariance (GRCov), has the additional characteristic of being robust to price jumps. It applies the MedRV of Andersen et al. (2010) for variance estimation and the Gaussian rank correlation for correlation estimation. Previous proposals for jump robust covariance estimation, were made by Barndorff-Nielsen and Shephard (2004), Boudt et al. (2008) and Mancini and Gobbi (2009). Compared to these estimators, the GRCov has the advantage that it is guaranteed to be positive semidefinite and computationally simple, also in high dimensions. Furthermore, it can make a more efficient use of the available high-frequency data because of the flexibility to use a different sampling frequency for variance and correlation estimation. The simulation study confirms that the GRCov is competitive with the existing estimators in small dimensions and becomes preferable as the dimension grows. The gains in accuracy of disentangling the estimation of covariance into its variance and correlation components are shown to be substantial for non-synchronous price processes. In an application to portfolio allocation on the Dow Jones Industrial Average constituents, we find that the GRCov leads to more stable portfolios with a lower risk.

The remainder of the paper is structured as follows. In the next section we introduce the Gaussian Rank Covariance. In Section 3, we compare the performance of the proposed estimator to other robust estimators in a Monte Carlo simulation. Subsequently, we analyze the properties of these estimators when applied on real data and investigate their performance in a medium-sized portfolio optimization problem. Finally, Section 5 summarizes our main findings and points out some directions for future research.

2 Robust covariance estimation

2.1 General framework

The proposed covariance estimator uses high-frequency price series to estimate the daily covariance matrix. The prices are assumed to be generated by a continuous-time log-price process belonging to the class of Brownian SemiMartingale with Finite Activity Jumps (BSMFAJ) models. Suppose that there are N assets and write $p(s)$ the vector process containing the log-prices of these N assets. Under the BSMFAJ model, the process $p(s)$ has a drift component, a Brownian martingale component with time-varying volatility and a jump component. Let s denote time and $\mu(s)ds$ the value of the drift at time s. Denote by $\Omega(s)$ the $N \times N$ càdlàg process such that $\Sigma(s) = \Omega(s)\Omega'(s)$ is the spot covariance matrix process of the continuous component of the price diffusion. Let $w(s)$ be a vector of N independent Brownian motions. The jump process depends on an N-dimensional finite activity counting process $q(s)$ whose change indicates the number of jump occurrences at time s. The $N \times N$ process $K(s)$ controls the magnitude of jumps such that $K(s)dq(s)$ is the contribution of the jump process to the price diffusion at time s. We then have that the N-dimensional log-price diffusion can be decomposed as follows:

$$\text{BSMFAJ:} \quad dp(s) \;=\; \mu(s)ds + \Omega(s)dw(s) + K(s)dq(s). \tag{2.1}$$

Our object of interest is the integrated covariance matrix (ICov) over the interval $[0,1]$, defined as

$$\text{ICov} = \int_0^1 \Sigma(s)ds. \tag{2.2}$$

Throughout the paper, the time is rescaled such that the interval $[0,1]$ corresponds to one day. In order to consistently estimate the ICov in the presence of jumps, an estimator robust to jumps, is required. First we rewrite the ICov by splitting the spot covariances into spot variances and spot correlations. Therefore, define the spot correlation between the log-prices of assets k and l as follows

$$\rho_{(kl)}(s) = \frac{\Sigma_{(kl)}(s)}{\sqrt{\Sigma_{(kk)}(s)\Sigma_{(ll)}(s)}}, \tag{2.3}$$

with $\Sigma_{(kl)}(s)$ the element (k,l) of the spot covariance matrix $\Sigma(s)$. This implies that

$$\Sigma_{(kl)}(s) = \rho_{(kl)}(s)\sigma_{(k)}(s)\sigma_{(l)}(s), \tag{2.4}$$

with $\sigma_{(k)}(s) = \Sigma_{(kk)}^{\frac{1}{2}}(s)$. Under smoothness conditions on $\Sigma(s)$, we can approximate the element (k,l) of the ICov as follows

$$\mathrm{ICov}_{(kl)} = \int_0^1 \rho_{(kl)}(s)\sigma_{(k)}(s)\sigma_{(l)}(s)ds \tag{2.5}$$

$$\approx \delta \sum_{i=1}^{\lfloor 1/\Delta \rfloor} \sum_{j=1}^{\lfloor 1/\delta \rfloor} \rho_{(kl)}((i-1)\Delta)\ \sigma_{(k)}((j-1)\delta)\ \sigma_{(l)}((j-1)\delta)\ \mathrm{I}(\tau_{i,j} > 0). \tag{2.6}$$

Equation (2.6) is the discretized version of (2.5), with Δ and δ the sampling frequencies for the spot correlation and variance respectively. Denote $r_{i,\Delta} = p(i\Delta) - p((i-1)\Delta)$ and $r_{j,\delta} = p(j\delta) - p((j-1)\delta)$ the returns sampled at the frequency Δ and δ, for $i = 1, \ldots, \lfloor 1/\Delta \rfloor$ and $j = 1, \ldots, \lfloor 1/\delta \rfloor$.[1] In (2.6), $\mathrm{I}(\cdot)$ stands for the indicator function and $\tau_{i,j}$ for the overlap in time between the intervals spanned by the returns $r_{i,\Delta}$ and $r_{j,\delta}$ given by

$$\tau_{i,j} = \max(0, \min(i\Delta, j\delta) - \max((i-1)\Delta, (j-1)\delta)).$$

The element $\mathrm{I}(\tau_{i,j} > 0)$ in (2.6) ensures that spot correlations and variances are only multiplied for matching intervals, allowing δ and Δ to be different.

It follows naturally to estimate the integrated covariance using (2.6) where the spot variance and correlation are replaced with local, jump robust estimators:

$$\widehat{\mathrm{ICov}}_{(kl)\Delta,\delta} = \delta \sum_{i=1}^{\lfloor 1/\Delta \rfloor} \sum_{j=1}^{\lfloor 1/\delta \rfloor} \hat{\rho}_{(kl)i\Delta}\ \hat{\sigma}_{(k)j\delta}\ \hat{\sigma}_{(l)j\delta}\ \mathrm{I}(\tau_{i,j} > 0). \tag{2.7}$$

The practical implementation of this estimator requires the choice of a local volatility and correlation estimator as well as the sampling frequency of the returns used in the estimation.

[1]The function $\lfloor \cdot \rfloor$ returns the largest integer less than or equal to its argument.

2.2 Spot volatility estimation

We estimate the spot volatility in (2.6) by the square root of a jump robust estimator of the integrated variance computed over a local window of E observations of δ-sampled returns. For the sake of simplicity and without loss of generality, we take the same sampling frequency δ and the same local window length E for every asset. We discuss the choice of sampling frequency and length of local window in Subsection 2.4. Several propositions have been made over the years to estimate the integrated variance (see Patton and Sheppard (2009) for a general review). We opt for the MedRV estimator proposed by Andersen et al. (2010), because of its computational simplicity and high robustness to both zero returns and outliers due to price jumps.

Denote by $r_{(k)j,1},...,r_{(k)j,E}$ the set of observations centered around $r_{(k)j,\delta}$, the jth intraday return of asset k.[2] For each local window we thus estimate $\sigma^2_{(k)j\delta}$ by

$$\hat{\sigma}^2_{(k)j\delta} = \mathrm{c}\delta^{-1}\frac{E}{E-4}\sum_{e=3}^{E-2}\mathrm{med}(|r_{(k)j,e-2}|,\ldots,|r_{(k)j,e+2}|)^2. \tag{2.8}$$

The correction factor $c = 1.624$ ensures that $c\delta^{-1}$ times the median of five contiguous squared returns is an unbiased estimator for spot volatility under the BSMFAJ model (see Teichroew (1956) and Andersen et al. (2010)). This is a slightly modified version of the original MedRV estimator, which takes the median of three contiguous squared return. This was shown by Christensen et al. (2010) not to be robust to additive outliers in the price series, caused e.g. by data input errors. In the presence of microstructure noise, the MedRV should be computed on preaveraged data, as explained in Andersen et al. (2010). Asymptotic properties for a class of related estimators of local variance in absence of jumps can be found in Kristensen (2010).

[2] This set spans the returns $r_{(k)g,\delta}$ with index g in the interval $[j-\frac{E-1}{2}, j+\frac{E-1}{2}]$. At the borders, when $j \le \frac{E-1}{2}$ the interval is given by $[1,E]$ or when $j > \frac{1}{\delta}-\frac{E-1}{2}$ the interval is given by $[\frac{1}{\delta}-E,\frac{1}{\delta}]$, with $\frac{1}{\delta}$ the number of observations for asset k within a day. We always take E an odd integer.

2.3 Spot correlation estimation

For robust estimation of the spot correlation coefficient in (2.6), using the returns sampled at the frequency $\Delta > \delta$, we proceed in two steps. In a first step, each return is standardized by dividing it by its estimated volatility. The spot correlation estimate $\hat{\rho}_{(kl)i\Delta}$ is then the correlation between the components k and l of A standardized returns in a local window around $r_{i,\Delta}$, with A such that it is reasonable to assume that correlation remains constant in the local window.

In the simulation study, we consider the correlation estimators associated with currently available jump robust covariance estimators, as well as a correlation estimator based on ranks, called the Gaussian rank correlation. The latter is calculated as follows. First, we compute the ranks of each component of the standardized returns. Denote by $g_{(k)i,a}$ the rank of the ath standardized return in the window centered around $r_{(k)i,\Delta}$. Subsequently, we obtain the corresponding Gaussian scores, also called the Van der Waerden scores, by plugging these ranks in the quantile function Φ^{-1} of the standard normal distribution:

$$z_{(k)i,a} = \Phi^{-1}\left(\frac{g_{(k)i,a}}{A+1}\right). \tag{2.9}$$

The Gaussian Rank Correlation matrix (GRCor) is the conventional correlation matrix of these scores, with element k, l given by

$$\mathrm{GRCor}_{(kl)i,\Delta} = \frac{\sum_{a=1}^{A} z_{(k)i,a} z_{(l)i,a}}{\sqrt{\sum_{a=1}^{A} z_{(k)i,a}^2 \sum_{a=1}^{A} z_{(l)i,a}^2}}, \tag{2.10}$$

and serves as the estimator $\hat{\rho}_{(kl)i\Delta}$ in (2.7). If the number of observations in the local window tends to infinity, the GRCor converges to the spot correlation. The main advantages of this estimator are its high robustness to jumps, computational simplicity even in high dimensions, and its positive semidefiniteness. Moreover, this estimator is asymptotically as efficient as the usual Pearson correlation estimator (Hájek and Sidak, 1967).

2.4 Gaussian rank covariance

Combining the MedRV for the spot volatility estimation and the GRCor for the spot correlation, yields the Gaussian rank covariance estimator for the daily integrated covariance matrix:

$$\mathrm{GRCov}_{\Delta,\delta} = \delta \sum_{i=1}^{\lfloor 1/\Delta \rfloor} \sum_{j=1}^{\lfloor 1/\delta \rfloor} D_{j,\delta} \mathrm{GRCor}_{i,\Delta} D_{j,\delta} \mathrm{I}(\tau_{i,j} > 0), \tag{2.11}$$

where $D_{j,\delta} = \mathrm{diag}(\hat{\sigma}_{(1)j\delta}, \ldots, \hat{\sigma}_{(N)j\delta})$ is the diagonal matrix containing the MedRV spot volatility estimates and $\mathrm{GRCor}_{i,\Delta}$ is the Gaussian rank correlation matrix having (2.10) as the k, lth element. Since GRCor is positive semidefinite and the MedRV estimates are always positive, the GRCov is guaranteed to be positive semidefinite.

Sampling frequency: In the absence of market frictions and for perfectly synchronized prices, the higher the sampling frequency, the more precise the ex post covariance estimate based on the high-frequency data will be. Due to asynchronicity however, correlations measured using ultra high frequency returns are typically biased towards zero, which is known as the Epps effect. As shown by Zhang (2009), the bias shrinks when market liquidity increases. On a sample of stock price data of 1971, Epps (1979) found that correlations only stabilize when sampling at 3 hour frequency. Tóth and Kertész (2006) document that because of an increase in market liquidity, the stabilizing frequency is higher for more recent data. However, it is still much lower than optimal sampling frequencies for realized volatility measures. Volatility signature plots indicate that realized volatility measures stabilize when sampling at about 2 minutes (Andersen et al., 2010). We will therefore always take $\Delta > \delta$. In the simulation study we use the 30-second frequency for the spot variance estimation and consider estimation of correlation using the 30-second, 1- and 5-minute frequency.

Window length: Previous studies pointed out that correlations are time-varying, but are highly persistent and the changes in correlation on an intraday scale are often negligible (Tang, 1995). In the simulation study and empirical application, we take a local window of one day to estimate correlation. Variances change faster over time and even exhibit intraday patterns (Andersen and Bollerslev, 1997). Therefore, it is useful to estimate spot variance and spot correlation using distinct window lengths. The width of the local window represents a trade-off. For a given sampling frequency, a small (large) local window will yield estimates of the spot volatility with a small (large) bias but large (small) variance. We set the length of local window to 15 minutes around a return.

3 Monte Carlo simulation

Through a Monte Carlo study, we assess the efficiency gains of (i) using the GRCor rather than another correlation matrix estimator, (ii) estimating the spot variance and spot correlation at different frequencies. For the latter, we make a comparison with previously proposed (jump robust) covariance matrix estimators. Barndorff-Nielsen and Shephard (2004) were the first to introduce a jump robust covariance estimator, called the Realized Bi-Power Covariation (RBPCov). To overcome some issues the RBPCov faces such as lack of positive semidefiniteness and a relatively large finite sample bias in the presence of jumps, Boudt et al. (2008) proposed the Realized Outlyingness Weighted Covariance (ROWCov). The main disadvantage of this last estimator is that it suffers from a curse of dimensionality, which makes it not suitable in practice for large-scale portfolio optimization. Finally, we consider the threshold covariance estimator (THRESCov). The precise definition of these estimators is described in Appendix. Before addressing the above questions, we first describe our simulation setup.

Simulation design: As in Barndorff-Nielsen et al. (2010), for each second of each day[3], we generate hypothetical prices, with $p_{(k)}(s)$ the associated log-price of asset k, from the log-price diffusion given by

$$\begin{aligned} dp_{(k)}(s) &= \mu(s)ds + dV_{(k)}(s) + dF_{(k)}(s) + J_{(k)}(s) \\ dV_{(k)}(s) &= \rho\ \sigma_{(k)}(s)\ dB_{(k)}(s) \\ dF_{(k)}(s) &= \sqrt{1-\rho^2}\ \sigma_{(k)}(s)\ dW(s) \\ J_{(k)}(s) &= \kappa(s)\ dq_{(k)}(s) \end{aligned} \tag{3.1}$$

with $k = 1, \ldots, N$. This diffusion process consists of 4 main components: μ represents the constant drift of the process, $dV_{(k)}$ and $dF_{(k)}$ denote the individual and common factor respectively and $J_{(k)}$ yields the jumps in the price process. The components of B are independent Brownian motions. W stands for a standard Brownian motion scaled by $\sqrt{1-\rho^2}$ to determine the strength of the common factor. The random spot volatility is given by $\sigma_{(k)}(s) = \exp(\beta_0+\beta_1\varphi_{(k)}(s))$, with $d\varphi_{(k)}(s) = \alpha\varphi_{(k)}(s)ds+dB_{(k)}(s)$, for $k = 1, \ldots, N$. The correlation between the changes in $p_{(k)}$ and $p_{(l)}$ equals then $1-\rho^2$ for $k \neq l$.

We calibrate the parameters $(\mu, \beta_0, \beta_1, \alpha, \rho)$ at $(0.03,\ -5/16,\ 1/8,\ -1/40,\ -0.3)$ as in Barndorff-Nielsen et al. (2010). The stationary distribution of φ is utilized to restart the process each day at $\varphi(0) \sim N(0, (-2(\beta)^2/\alpha)^{-1})$. The jump occurrences are governed by the Poisson process $q(s)$ with constant intensity $\kappa^*/23400$ with κ^* the expected daily number of jumps per asset. We model the size of the jumps $\kappa(s)$ as the product between the realization of a uniformly distributed random variable on

[3]Hence, 23400 observations per day, assuming a market that is open 6.5 hours a day such as the NYSE.

$([-2,-1]\cup[1,2])/\sqrt{2\kappa^*}$ and the mean value of the stochastic volatility process $\sigma_{(k)}$ of that day, for $k = 1, \ldots, N$. Note that the lower the intensity of the jump process, the larger the jumps are.

We consider both synchronized and non-synchronous transaction times. For the latter, we use independent Poisson sampling schemes such that the inter transaction times are exponentially distributed with on average one transaction every 5 seconds. We align the price series to a regular grid, using the previous tick approach. We consider 30-second, 1- and 5-minute returns.

<u>Performance measure</u>: The accuracy of the different estimators of the daily covariance matrix is measured by means of the Frobenius distance between the true and estimated daily integrated covariance matrix:

$$\sum_{1\leq i,j\leq N} (\mathrm{ICov}_{(i,j)} - \widehat{\mathrm{ICov}}_{(i,j)})^2,$$

averaged over 1000 simulation runs.

<u>Choice of correlation estimator</u>: Many estimators can be constructed using the strategy in (2.6), i.e. disentangling covariance estimation into spot variance and correlation estimation. To restrict the focus, we fix the choice of the spot variance estimator to the MedRV implemented with 30-second returns. In this paper, we propose to estimate the correlation using the GRCor in (2.10). We compare this method with the use of the correlation estimators associated to the Realized Covariance, the Realized Bi-Power Covariation, the Realized Outlyingness Weighted Covariance and the Threshold Covariance. We call the resulting covariance estimators the MedRV-RCor, MedRV-RBPCor, MedRV-ROWCor and MedRV-THRESCor respectively.

Table 1 reports the Frobenius distance between the true and the estimated covariance matrix in basis points for various sampling frequencies, dimensions $N = 5$ and

$N = 30$ and the case of synchronous and non-synchronous transactions. Table 1 also reports the percentage of positive definite MedRV-RBPCor estimates. Note that all other estimators are positive semidefinite by construction.

Consider first the case of synchronous observations. Our simulation results confirm the well known fact that in the absence of microstructure noise, higher sampling frequencies yield more precise covariance estimates. This holds for all estimators, with the MedRV-RCor being the most precise in the absence of jumps. In the presence of jumps, however, the estimator based on the RCor loses a lot of precision and all robust estimators perform better. The relative precision of the robust estimators seems to depend on the specific setting. For $N = 5$ the MedRV-ROWCor often outperforms the other estimators. But, the relative efficiency of the MedRV-ROWCor deteriorates when the dimension grows: for $N = 30$ the MedRV-ROWCor is outperformed by most other robust estimators at the lower sampling frequencies. Another well performing robust estimator, the MedRV-RBPCor, suffers from a severe lack of positive semidefiniteness for $N = 30$, in particular at lower sampling frequencies. The remaining two estimators, the MedRV-GRCor and the MedRV-THRESCor, deliver comparable results.

Let us now study the results in case the transactions for the different assets are observed asynchronously. It no longer holds that the higher the sampling frequency the better. This finding is consistent with Zhang (2009), who shows that the non-synchronicity bias increases with the sampling frequency. For the MedRV-RCor the sampling frequency represents a trade-off between accuracy and the non-synchronicity bias. At high sampling frequencies the estimate has a low variance but it is largely biased due to the Epps effect. At lower sampling frequencies, the bias due to the Epps effect is smaller but the estimates have a higher variance. We find that the 1-minute or 5-minute sampling frequency is best for all estimators. If there is asynchronicity, the optimal sampling frequency in the absence of jumps is the lowest one considered here

(5 minutes). In the presence of jumps, optimal sampling frequencies seem to be a bit higher (typically 1 minute), because of the trade-off between the bias caused by jumps and the bias caused by the Epps effect. Indeed, the percentage of returns affected by jumps decreases with the sampling frequency, while the Epps effect is more pronounced at high frequencies. The introduction of asychronicity does not alter the conclusions regarding the robustness properties of the different estimators considered. For $N = 5$ the MedRV-ROWCor outperforms the other robust estimators, while for $N = 30$ the most precise estimator depends on the specific setting. Note that the lack of positive semidefiniteness of the MedRV-RBPCor for $N = 30$ becomes even more problematic than in case of synchronous observations.

Disentangling covariance estimation: An interesting remaining question is whether the disentangling of spot covariance estimation into separate estimation of spot correlation and spot variances at different sampling frequencies improves the covariance estimation. We compare the MedRV-RBPCor, MedRV-ROWCor, MedRV-THRESCor with RBPCov, ROWCov and THRESCov, respectively.

The gains of disentangling the estimation of variance and correlation are most substantial for non-synchronous transactions. For $N = 5$ and in the absence of jumps the best sampling frequency (of the ones considered in the simulation setting) is 5 minutes for the "disentangled estimators" and 1 minute for the traditional estimators. Let us focus on $\Delta = 5$ minutes. We see from Table 1 that the average Frobenius distances range from 0.9 to 0.12 for the disentangled estimators, compared to 0.40 to 0.53 for the other estimators. This means that precision can be more than tripled by disentangling the estimation of variance and correlation. For $N = 5$, and in presence of jumps, we see again that the disentangled estimators MEDRV-RBPCor, MEDRV-ROWCor and MedRV-THRESCor outperform the corresponding RBPCov, ROWCOv, and THRESCov estimators. Similar conclusions can be made for $N = 30$. There is an enormous gain using the disentangled estimators instead of their counterparts estimating spot variance and correlation at the same frequency. Note however that at frequencies 1 minute and 30 seconds, the advantage of disentangling is less clear-cut. Nevertheless, in almost all cases the average Frobenius distances are much higher than for Δ=5 minutes.

For synchronous transactions, where the Epps-effect does not play, the best results are obtained at the highest frequency, i.e. Δ=30 seconds. At this high frequency, there is little or no advantage of disentangling variance and correlation estimation. Note, however, that at the other sampling frequencies, also for synchronous observations, the disentangled estimators outperform the other ones.

In conclusion, the simulation study has shown the usefulness of the GRCov and MedRV-THRESCor in the presence of non-synchronous observations. Their robustness to jumps is competitive to the existing robust estimators. We have also shown that the benefits of disentangling the estimation of covariance into its variance and correlation components are substantial.

Table 1: Average Frobenius distance between estimated and true ICov.

		$N=5$						$N=30$					
		synchronous			**non-synchronous**			**synchronous**			**non-synchronous**		
	$\Delta \setminus \kappa^*$	0	1	5	0	1	5	0	1	5	0	1	5
MedRV-RCor	30-sec	0.06	2.30	2.62	0.43	3.20	3.62	1.93	94.71	114.20	17.85	133.26	157.70
MedRV-RBPCor	30-sec	0.06	0.08	0.09	0.42	0.50	0.53	2.03	2.38	3.10	17.45	19.55	23.29
(%psd)	30-sec	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(0.94)	(0.99)	(1)	(0.17)	(0.50)
MedRV-ROWCor	30-sec	0.06	0.07	0.08	0.43	0.43	0.36	1.93	2.01	2.31	17.89	16.98	15.50
MedRV-THRESCor	30-sec	0.06	0.07	0.07	0.50	0.49	0.41	2.06	2.07	2.22	20.26	18.99	17.52
GRCov	*30-sec*	*0.06*	*0.07*	*0.08*	*0.43*	*0.48*	*0.54*	*1.93*	*2.00*	*2.87*	*17.89*	*18.70*	*23.66*
RCov	30-sec	0.04	10.07	5.64	0.32	10.36	5.91	1.30	60.48	37.26	13.54	72.30	49.59
RBPCov	30-sec	0.05	0.23	0.49	0.40	0.33	0.28	1.67	6.23	13.17	16.51	10.48	6.63
(%psd)	30-sec	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(0.94)	(0.99)	(1)	(0.17)	(0.50)
ROWCov	30-sec	0.04	0.04	0.05	0.32	0.33	0.31	1.30	1.30	1.66	20.68	19.52	18.16
THRESCov	30-sec	0.06	0.06	0.06	0.51	0.49	0.45	1.82	1.58	1.82	19.94	18.37	18.83
MedRV-RCor	1-min	0.06	2.29	2.64	0.20	2.80	3.22	1.97	95.08	115.23	7.99	116.67	140.17
MedRV-RBPCor	1-min	0.06	0.09	0.12	0.21	0.28	0.36	2.05	2.66	4.64	8.12	10.40	15.68
(%psd)	1-min	(1)	(1)	(1)	(1)	(0.99)	(1)	(1)	(0.16)	(0.43)	(1)	(0.02)	(0.02)
MedRV-ROWCor	1-min	0.06	0.07	0.08	0.20	0.21	0.16	1.98	2.01	19.09	8.24	7.64	29.46
MedRV-THRESCor	1-min	0.07	0.08	0.07	0.26	0.26	0.21	2.17	2.12	2.23	9.79	9.02	8.61
GRCov	*1-min*	*0.06*	*0.08*	*0.14*	*0.21*	*0.26*	*0.41*	*1.98*	*2.25*	*5.70*	*8.10*	*9.70*	*18.06*
RCov	1-min	0.08	10.19	5.73	0.15	10.26	5.82	2.56	64.45	41.27	5.80	67.37	44.59
RBPCov	1-min	0.10	0.41	0.85	0.19	0.32	0.55	3.30	11.75	23.36	6.92	7.41	11.08
(%psd)	1-min	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(0.13)	(0.41)	(1)	(0)	(0.02)
ROWCov	1-min	0.08	0.08	0.09	0.15	0.15	0.15	2.59	2.53	13.84	7.36	6.74	17.36
THRESCov	1-min	0.10	0.10	0.10	0.25	0.24	0.24	3.32	2.90	3.49	9.03	8.06	9.42
MedRV-RCor	5-min	0.07	2.39	2.73	0.09	2.59	2.95	2.11	99.08	118.04	2.99	107.73	127.71
MedRV-RBPCor	5-min	0.07	0.17	0.51	0.10	0.24	0.66	2.36	5.98	22.27	3.32	8.79	29.05
(%psd)	5-min	(1)	(0.85)	(0.99)	(1)	(0.83)	(0.98)	(0)	(0)	(0)	(0)	(0)	(0)
MedRV-ROWCor	5-min	0.07	0.08	0.47	0.09	0.10	0.55	2.18	44.80	97.56	3.19	49.31	106.55
MedRV-THRESCor	5-min	0.08	0.09	1.10	0.12	0.14	1.30	2.49	2.78	50.42	3.94	4.51	58.73
GRCov	*5-min*	*0.07*	*0.19*	*0.93*	*0.10*	*0.29*	*1.12*	*2.16*	*7.03*	*41.46*	*3.46*	*11.06*	*49.92*
RCov	5-min	0.40	10.88	6.66	0.41	10.88	6.66	12.87	96.73	72.13	12.86	96.87	72.61
RBPCov	5-min	0.52	1.76	2.85	0.53	1.72	2.76	16.47	49.37	65.72	16.64	46.02	61.41
(%psd)	5-min	(0.99)	(0.88)	(0.98)	(0.99)	(0.84)	(0.98)	(0)	(0)	(0)	(0)	(0)	(0)
ROWCov	5-min	0.40	0.43	2.17	0.40	0.44	2.16	16.21	67.45	69.99	17.36	67.48	69.62
THRESCov	5-min	0.48	0.46	1.31	0.50	0.49	1.37	15.28	16.31	35.66	15.80	17.10	37.97

Note: We report the Frobenius distance in basis points between the integrated covariance matrix and the estimates over 1000 Monte Carlo replications. κ^* indicates the expected number of jumps per asset per day. Prices are observed either synchronously or asynchronously. The spot volatility is estimated based on the 30-sec sampling frequency and the spot correlation on the sampling frequencies Δ indicated in the Table. The row (%psd) reports the percentage of positive semidefinite estimates for the RBPCor.

4 Empirical application

Accurate predictions of the covariance matrix are a critical input for portfolio allocation. Fleming et al. (2003), De Pooter et al. (2008), Bandi et al. (2008) and Liu (2009) showed the superiority of portfolios based on covariance forecasts using high-frequency data instead of daily returns, to measure the daily realized variability. We study here the portfolio performance gains obtained by using the robust GRCov and MedRV-THRESCov in comparison to the standard RCov for minimum variance portfolio allocation during the credit crisis.

We expect that a minimum variance portfolio allocation strategy based on the RCov will tend to give significantly lower weights to assets that have jumped on the preceding day. Since jumps tend to be less persistent than smooth price variation (Andersen et al., 2007), this might induce an overreaction to realized jump risk. Therefore, more stable portfolio weights may be obtained using a robust estimator as input in the portfolio allocation.

Data and investment strategy: The investment universe covers 27 of the 30 Dow Jones Industrial Average constituents at the beginning of 2008.[4] The data sample consists of the intraday transaction prices from the Trade and Quotes database (TAQ) of the New York Stock Exchange and contains 625 trading days ranging from July 2, 2007 to December 31, 2009.[5] The portfolio allocation proceeds in two steps.

First, one-day ahead forecasts of the conditional covariance matrix $\hat{\Sigma}_t$ of daily returns are constructed. We follow Fleming et al. (2001, 2003), De Pooter et al. (2008) and Bannouh et al. (2009) by specifying $\hat{\Sigma}_t$ as an exponentially weighted average of its

[4]Tickers of the stocks in the sample are: AA, AXP, BA, C, CAT, DD, DIS, GE, HD, HON, HPQ, IBM, INTC, JNJ, JPM, KO, MCD, MMM, MO, MRK, MSFT, PFE, PG, UTX, VZ, WMT, XOM. Because of too many missing observations AIG, GM, T were removed from our sample.

[5]We removed half trading days from our sample, namely 2007-11-23, 2007-12-24, 2008-07-03, 2008-11-28, 2008-12-24, 2009-11-27, 2009-12-24. Prior to the analysis, the data is cleaned using the step-by-step procedure proposed in Barndorff-Nielsen et al. (2009) and implemented in the R package RTAQ (Cornelissen and Boudt, 2010)

lagged value and the realized variability of the previous day:

$$\hat{\Sigma}_t = \exp(-\alpha)\hat{\Sigma}_{t-1} + \alpha \exp(-\alpha)(V_{t-1} + \eta_{t-1}\eta_{t-1}), \tag{4.1}$$

where α is the decay parameter, V_{t-1} is an estimate of the integrated covariance matrix on day $t-1$ and η_{t-1} is the close-to-open return between day $t-2$ and day $t-1$.

Second, the optimal weights are determined for a pure volatility timing investment strategy. We consider a minimum variance investor subject to a long only weight constraint and an upper position limit of 20%.[6] Consequently, for each day t he solves the following optimization problem:

$$\min_{w_t} \quad w_t'\hat{\Sigma}_t w_t \tag{4.2}$$
$$\text{subject to} \quad w_t'\iota = 1,\ \min(w_t) \geq 0 \ \text{ and } \ \max(w_t) \leq 0.2,$$

with w_t the $N \times 1$ vector of portfolio weights and ι a $N \times 1$ vector of ones.

Our goal is to evaluate the effect of the estimator of V_t on the portfolio performance. We compare the common choice for V_t, the RCov, with the proposed GRCov and MedRV-THRESCor. The latter 2 estimators performed best in the simulation study in Section 3. Note that the solution to the quadratic programming problem in (4.2) requires V_t to be positive semidefinite, which rules out the RBPCov and the MedRV-RBPCor. As recommended by Andersen et al. (2010), we use the 2-minute sampling frequency to compute the spot variance components in the GRCov and MedRV-THRESCor. The RCov and the correlation components in GRCov and MedRV-THRESCor are based on returns sampled at the 2, 5, 10, 15, 30 or 65 minute

[6] This contrasts with Fleming et al. (2001), who use ex post means to construct a mean-variance optimal investment strategy. Since expected returns are not the same as realized returns, this approach biases the optimal portfolio to assets with high ex post returns even though an optimal forward looking portfolio might not hold many of these assets.

frequency using previous tick interpolation.

A final practical question is the calibration of the decay parameter α in the forecasting model (4.1). We estimate α either by maximum likelihood (statistically optimal), assuming the returns to be conditionally normal with conditional covariance matrix Σ_t, or as the decay parameter for which the investment strategy has the lowest standard deviation (economically optimal). We calibrate α for all strategies to the value that optimizes the performance of the method using the RCov over the full sample.

Performance measures: We assess the performance of the different investment strategies by three evaluation criteria: the standard deviation and the mean of daily portfolio returns, and the average portfolio turnover. The daily portfolio turnover is defined as the percentage of wealth traded that day: $\mathrm{TO}_t = |w_t - w_{t+}|'\iota$, where w_t is vector of weights at the rebalancing period t and w_{t+} is the vector of weights before rebalancing at t.

Results: Table 2 reports the portfolio performance measures for the RCov, GRCov and MedRV-THRESCor strategies, for various sampling frequencies and for the economically and statistically optimal α. The economically optimal estimate of α is 0.22, putting a higher weight on the lagged V_t in (4.1) than the maximum likelihood approach for which α varies between 0.07 (using 2-minute returns) and 0.03 (using 65-minute returns). For both the statistically and economically optimal α, and for all estimators considered, we find that the 30-minute sampling frequency yields optimal portfolios with the lowest standard deviation, confirming De Pooter et al. (2008).

Differences between the considered estimators in terms of annualized standard deviation are quite small. At the optimal 30-minute sampling frequency the GRCov does yield the lowest annualized standard deviation, both for the statically and economically optimal α.

Since investors faced the worst stock market conditions in decades (and we restrain

Table 2: Portfolio performance of the minimum variance strategy based on the RCov, GRCov and MedRV-THRESCor.

	Statistically optimal α						**Economically optimal α**					
sampling freq	2 min	5 min	10 min	15 min	*30 min*	65 min	2 min	5 min	10 min	15 min	*30 min*	65 min
α	0.07	0.05	0.04	0.04	*0.03*	0.03	0.22	0.22	0.22	0.22	*0.22*	0.22
	RCov											
St. dev.	23.05	23.06	22.94	22.83	*22.29*	24.11	22.61	23.02	22.71	22.75	*22.12*	22.18
Mean	-14.17	-14.01	-12.55	-12.94	*-12.45*	-15.84	-13.42	-13.96	-13.11	-13.72	*-14.11*	-14.48
TO	13.88	11.46	10.08	11.27	*10.49*	12.00	35.90	41.32	47.13	50.88	*55.96*	65.96
	GRCov											
St. dev.	22.96	23.71	23.41	22.79	*22.19*	22.87	22.47	23.13	22.71	22.75	*21.99*	22.24
Mean	-13.91	-14.86	-14.25	-13.37	*-13.09*	-14.10	-12.68	-13.34	-13.27	-13.20	*-12.43*	-12.25
TO	14.30	11.25	9.37	10.12	*9.90*	10.86	36.52	38.80	41.80	44.47	*53.02*	64.70
	MedRV-THRESCor											
St. dev.	22.89	23.78	23.26	23.10	*22.69*	24.02	22.56	23.07	22.58	23.14	*22.53*	23.89
Mean	-13.74	-15.06	-13.99	-13.60	*-13.39*	-15.09	-12.82	-13.21	-12.75	-12.62	*-12.77*	-13.57
TO	14.58	11.39	9.43	10.21	*9.88*	10.35	37.45	39.26	42.87	44.98	*51.74*	60.08

Each column reports the portfolio performance measures for a certain sampling frequency and the corresponding decay parameter α, optimized in-sample for the RCov using the maximum likelihood (left panel) or minimum St. dev. criterion (right panel). St. dev. represents the annualized standard deviation of the daily portfolio returns in percentage points. Mean is the annualized average daily portfolio return in percentage points. TO is the annualized average daily turnover. For the equal-weight portfolio the St. dev., Mean and TO are 33.79, -15.42 and 3.5 respectively.

from short-selling), we find negative average returns for all our strategies. Nevertheless, the minimum variance portfolios all outperformed the equal-weight portfolio over our sample.

A convincing result in favor of the proposed GRCov is that its use tends to result in a significantly lower portfolio turnover. At the 30-minute sampling frequency, the average turnover of the RCov strategy is 55.96 and 10.4, while for the GRCov it is 53.02 and 9.9, for the economically and statistically optimal α respectively. On the turnover criterion, the MedRV-THRESCor performs even slightly better than the GRCov. Interestingly, for the turnover criterion, the optimal sampling frequencies are typically much higher than the 30-minute sampling frequency.

Sensitivity to the decay parameter α: Table 3 presents the performance measures for the three estimator strategies based on the 30-minute sampling, for α ranging from 0.01 to 0.28. As before, we find that differences in terms of annualized standard devia-

Table 3: Sensitivity of portfolio performance to the decay rate α.

α	0.01	0.04	0.07	0.10	0.13	0.16	0.19	*0.22*	0.25	0.28
	RCov									
St. dev.	23.06	22.31	22.44	22.35	22.22	22.16	22.13	*22.12*	22.13	22.16
Mean	-13.73	-12.79	-13.71	-13.90	-13.81	-13.95	-14.03	*-14.11*	-14.25	-14.41
TO	4.38	12.34	20.24	27.83	35.33	42.50	49.47	*55.96*	62.19	68.01
	GRCov									
St. dev.	22.91	22.24	22.44	22.45	22.33	22.18	22.06	*21.99*	21.93	21.90
Mean	-13.28	-13.29	-13.50	-13.54	-13.30	-13.05	-12.75	*-12.43*	-12.14	-11.89
TO	4.38	11.62	19.28	26.52	33.35	40.14	46.66	53.02	*59.15*	65.05
	MedRV-THRESCor									
St. dev.	23.21	22.72	22.92	22.93	22.79	22.65	22.57	*22.53*	22.52	22.53
Mean	-13.38	-13.65	-14.04	-14.00	-13.68	-13.42	-13.12	*-12.77*	-12.44	-12.17
TO	4.33	11.52	18.87	26.01	32.75	39.21	45.49	*51.74*	57.87	63.66

Each column reports the portfolio performance measures for a certain decay parameter α. The sampling frequency is 30 minutes. St. dev. represents the annualized standard deviation of the daily portfolio returns in percentage points. Mean is the annualized average daily portfolio return in percentage points. TO is the annualized average daily turnover. For the equal-weight portfolio the St. dev., Mean and TO are 33.79, -15.42 and 3.5 respectively.

tion are quite small between the different estimators. For increasing α the annualized mean return tends to decrease for the RCov, while it increases for the robust estimators. For large α, portfolios based on the GRCov yield the lowest annualized standard deviation and highest annualized return.

Table 3 also reveals that the value of the decay parameter α has a huge impact on the turnover. This decrease in turnover for smaller values of α is intuitive, since the closer α is to zero, the more smooth the conditional covariance estimates and hence the implied weights are. An important observation is that for a fixed value of α, the turnover is always lower for the robust estimators than for the RCov. The lower turnover of the robust estimator strategies can be attributed to the higher smoothness of these estimates. At e.g. the 30-minute sampling frequency, the average Frobenius distance over the sample period between GRCov_t and GRCov_{t-1}, is 17% lower than the distance between the RCov_t and RCov_{t-1}. The enhanced smoothness of the GRCov thus results in more stable portfolios over time, irrespective of the choice of α.

5 Conclusion

We propose a jump robust estimator for the daily covariance matrix, that exploits high-frequency intraday returns. An essential feature of our estimator, called the Gaussian rank covariance, is that it disentangles the estimation of spot covariance into the estimation of spot variances and spot correlations. Our estimator offers the following advantages: (1) it is robust with respect to non-synchronous transactions, (2) it is positive semidefinite, (3) it is robust with respect to jumps in the price level, (4) it remains accurate in high dimensions and (5) it is computationally simple. A simulation study revealed that the benefits of (1) are quite substantial and that the performance concerning (3) is competitive to existing estimators.

In an application to portfolio allocation on the universe of Dow Jones Industrial Average stocks we show the practical usefulness of the GRCov. Portfolios based on the GRCov are characterized by a lower turnover and usually by a lower risk, ceteris paribus, than those based on the standard Realized Covariance. In comparison to the estimators considered in the portfolio allocation applications of Fleming et al. (2003), De Pooter et al. (2008) and Liu (2009), the GRCov has the before mentioned advantages (1) and (3). Compared to the estimators suggested in Bandi et al. (2008) it has the additional advantage (2). With respect to the estimator proposed in Bannouh et al. (2009), it has advantage (3).

Finally, some interesting directions for future research remain open. An important first note is that the proposed estimator could easily be adjusted by existing procedures to account for microstructure noise, for example, by calculating the optimal sampling frequency for each diagonal element of the covariance matrix by the procedure proposed in Bandi et al. (2008). Also, the choice of the local window length to estimate the spot variances and correlation could be investigated. Another direction for future research is the inclusion of both jump robust covariance estimators and realized cojump variability in multivariate forecasting models.

Appendix: Description of competing estimators

The competing estimators are computed using the returns sampled at the frequency used to compute the correlation, i.e. returns sampled every Δ units of time. Suppose we want to estimate the integrated covariance matrix over $[0,1]$. The standard estimator is the Realized Covariance (RCov) defined as

$$\mathrm{RCov} = \sum_{i=1}^{\lfloor 1/\Delta \rfloor} r_{i,\Delta} r'_{i,\Delta}.$$

An alternative estimator, robust with respect to jumps, is the Realized Bi-Power Covariation (RBPCov), proposed by Barndorff-Nielsen and Shephard (2004). The element (k,l) of this matrix is defined as

$$\mathrm{RBPCov}_{(kl)} = \frac{\pi}{8}\Bigg(\sum_{i=2}^{\lfloor 1/\Delta \rfloor} \left| r_{(k)i,\Delta} + r_{(l)i,\Delta} \right| \left| r_{(k)i-1,\Delta} + r_{(l)i-1,\Delta} \right| - \left| r_{(k)i,\Delta} - r_{(l)i,\Delta} \right| \left| r_{(k)i-1,\Delta} - r_{(l)i-1,\Delta} \right| \Bigg),$$

where $r_{(k)i,\Delta}$ is the k-th component of the return vector $r_{i,\Delta}$. The main disadvantages of this estimator are that it can have a large finite sample bias when jumps affect contiguous returns and that the resulting covariance matrix is not ensured to be positive semidefinite.

To overcome these issues, Boudt et al. (2008) proposed the Realized Outlyingness Weighted Covariance (ROWCov) given by

$$\mathrm{ROWCov} = c_w \frac{\sum_{i=1}^{\lfloor 1/\Delta \rfloor} w_{i,\Delta} r_{i,\Delta} r'_{i,\Delta}}{\frac{1}{\lfloor 1/\Delta \rfloor} \sum_{i=1}^{\lfloor 1/\Delta \rfloor} w_{i,\Delta}}.$$

The weight $w_{i,\Delta}$ is one if the multivariate jump test statistic for $r_{i,\Delta}$ in Boudt et al. (2008) is less than the 99.9% percentile of the chi-square distribution with N degrees of freedom and zero otherwise. The scalar c_w is a correction factor ensuring consistency of the ROWCov for the ICov (2.2), under the BSMFAJ model. Advantages of the ROWCov compared to the RBPCov include a higher statistical efficiency, positive semidefiniteness and affine equivariance. However, the ROWCov suffers from a curse of dimensionality. Indeed, the ROWCov gives a zero weight to a return vector if at least one of the components is affected by a jump. In the case of independent jump occurrences, the average proportion of observations with at least one component being affected by jumps increases fast with the dimension of the series. This means that a potentially large proportion of the returns receives a zero weight, due to which the ROWCov can have a low finite sample efficiency in higher dimensions (see e.g. Table 1 in Alqallaf et al. 2009).

The threshold covariance matrix (THRESCov) proposed in Mancini and Gobbi (2009) is the next robust estimator we consider. Unlike the ROWCov, the THRESCov uses univariate jump detection rules to truncate the effect of jumps on the covariance estimate. As such, it remains feasible in high dimensions, but it is less robust to small cojumps. It can be computed as follows

$$\text{THRESCov}_{(kl)} = \sum_{i=1}^{\lfloor 1/\Delta \rfloor} r_{(k)i,\Delta} r_{(l)i,\Delta} 1_{\{r^2_{(k)i,\Delta} \leq \eta_{(k)\Delta}\}} 1_{\{r^2_{(l)i,\Delta} \leq \eta_{(l)\Delta}\}}.$$

The threshold value $\eta_{(k)\Delta}$ is set to $9\Delta^{-1}$ times the daily realized bi-power variation of asset k, as suggested in Jacod and Todorov (2009).

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